**ANGLO-CHINESE JUNIOR COLLEGE**

/100

**JC1 PROMOTIONAL EXAMINATION**

Higher 2

CANDIDATE

NAME

TUTORIAL/ INDEX

FORM CLASS NUMBER

**MATHEMATICS** **9758/01**

Paper 1 **1 October 2021**

**3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

|  |  |
| --- | --- |
| **Question** | **Marks** |
| **1** | **/3** |
| **2** | **/5** |
| **3** | **/5** |
| **4** | **/6** |
| **5** | **/7** |
| **6** | **/7** |
| **7** | **/8** |
| **8** | **/11** |
| **9** | **/11** |
| **10** | **/12** |
| **11** | **/12** |
| **12** | **/13** |

**READ THESE INSTRUCTIONS FIRST**

Write your index number, class and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.

Write your answers in the spaces provided in the Question Paper.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use an approved graphing calculator.

Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

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This document consists of **28** printed pages.

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**[Turn over**

1. State a sequence of transformations that will transform the curve with equation  on to the curve with equation , where is a positive constant. [3]

**2**  Solve algebraically the inequality . [3]

Hence solve the inequality . [2]

1. A curve *C* has equation

, , 

Show that . [2]

Hence, prove that curve *C* does not have any stationary point. [3]

















**4**

The diagram shows the curve. There are two vertical asymptotes with equations  and  respectively. The curve crosses the *x*-axis at the point *A* and has a maximum turning point at *B* where it crosses the *y*-axis.

The curve also has a minimum turning point at *C*. The coordinates of *A*, *B* and *C* are,  and  respectively, where *a*, *p* and *q* are constants.

Sketch the following curves and state the equations of the asymptotes, the coordinates of the turning points and of points where the curve crosses the axes, if any. Leave your answers in terms of *a*, *p* or *q* where necessary.

1. , and [3]
2. . [3]

**5** Referred to the origin *O*, points *A* and *B* have position vectors and  respectively. The modulus of  is 2 and  is a unit vector. The angle between and  is . Point *C* lies on *AB*, between *A* and *B*, such that, where 0 < *k <* 1.

**(i)** Express in terms of  and . [1]

**(ii)** Show that the length of projection of  is given by . [3]

**(iii)** Find, in terms of *k*, the area of triangle *OAC.*  [3]

1. The Cartesian equation of line  is , where  are constants.

The line is parallel to the vector . The line  passes through the origin and the point with position vector .

1. Given that  is perpendicular to, form an equation relating  [1]
2. Given that  intersects, show that  [3]
3. Hence express  [1]
4. Find the acute angle between  and. [2]

**7** The functions f and g are defined by

,

 where *k* is a constant.

1. Sketch on the same diagram the graphs of
2. 
3. 
4. 

stating the equations of any asymptotes and the coordinates of any endpoints. [3]

**(ii)** Find and state the domain of . [3]

1. Show that the composite function gf exists and find its range. [2]
2. The figure below shows a cross-section *OBCE* of a car headlight whose reflective surface is modelled in suitable units by the curve with parametric equations

, 

for , where *a* is a positive constant.

*B*

*C*

*O*

*E*

*T*

*S*

*x*

*y*

1. Find in terms of *a*
2. the length of *OE*, [2]
3. the maximum height of the curve *OBCE*. [1]
4. Show that . [3]

Point *B* lies on the curve and has parameter. *TS* is tangential to the curve at *B* and *BC* is parallel to the *x*-axis. Given that ,

1. show that . [2]
2. Show that the equation of normal to the curve at the point *B* is

,

where *k* is an exact constant to be determined. [3]

**9**  **(a)** Given that , find  in terms of *n*. [4]

**(b) (i)** Use the method of differences to show that  where *A* is a constant to be determined. [3]

**(ii)** Explain why the series  converges, and write down its value. [2]

**(iii)** Hence deduce that  is less than . [2]

1. Referred to the origin *O*, the points *A*, *B* and *C* have position vectors ,  and  respectively, where  and  are constants.
2. Given that *A, B* and *C* are collinear, show that *α* =5, and find the value of *β* . [3]

The plane  contains the line *L,*which has equation . The plane  is also parallel to the line that passes through the points *A* and *B*.

1. Find the shortest distance from point *A* to the line *L*. [2]
2. Show that the cartesian equation of the plane  is . [2]
3. Find the position vector of the foot of the perpendicular from point *A* to the plane . [3]
4. Hence find the reflection of the line that passes through points *A* and *B* about the plane. [2]
5. The figure below shows a container with an open top. The uniform cross section *ABCD* of the container is a trapezium with *AB* = *BC* = *CD* = 10 cm. *AB* and *CD* are each inclined to the line *BC* at an acute angle of  radians.

The length of the container is 50 cm and the container is placed on a horizontal table.

50 cm

*A*

*D*

*P*

*Q*

*S*





*B*

*C*

*R*

1. Show that the volume *V* of the container is given by

. [2]

Hence using differentiation, find the exact maximum value of *V*, proving that it is a maximum. [5]

**(ii)** For the remaining part of the question,  is fixed at .

Water fills the container at a rate of . At time *t* seconds, the depth of the water is *h* cm. The surface of the water is a rectangle *PQRS*. When  find the rate of change of

**(a)** the depth of the water, *h*, [3]

**(b)** the surface area of the water *PQRS*. [2]

**12** Mrs Tan plans to start a business which requires a start-up capital of $700,000. She decided to first save $200,000 by depositing money every month into a savings plan. For the remaining $500,000, she intends to take a loan from a finance company.

She deposited $3000 into the savings plan in the first month and on the first day of each subsequent month, she deposited $100 more than the previous month. Mrs Tan will continue depositing money into the savings plan until the total amount in her savings plan reaches $200,000. It is given that this savings plan pays no interest.

1. Find the month in which Mrs Tan’s monthly deposit will exceed $6,550. [2]
2. Find the number of months that it will take for Mrs Tan to save $200,000 and hence find the amount that she would have deposited in the last month. [4]

After Mrs Tan has saved $200,000, she took a loan of $500,000 from a finance company. To repay the loan from the finance company, Mrs Tan would have to pay a monthly payment of  at the beginning of each month, starting from the first month. An interest of  per month will be charged on the outstanding loan amount at the end of the month.

1. Show that the outstanding amount at the end of  month, after the interest has been charged, is , where *A* and *B* are exact constants to be determined. [3]
2. Find the amount of $*x*, to 2 decimal places, if Mrs Tan wants to fully repay her loan in 8 years. [2]
3. Using the value of *x* found in part **(iv)**, calculate the total interest that the finance company will earn from Mrs Tan at the end of 8 years. [2]